Module 5 - Decrease and Conquer Algorithm

1. Understanding Decrease and Conquer Approach

Concepts:

Definition: A problem-solving approach that exploits the relationship between a solution to a given instance of a problem and a solution to its smaller instance.

Process:

Decrease/Reduce: Reduce the problem instance to a smaller instance of the same problem.

Conquer/Solve: Solve the smaller instance of the problem.

Extend/Combine: Extend the solution of the smaller instance to obtain the solution to the original problem.

Comparison with Divide and Conquer:

Divide and Conquer: Divides the problem into multiple subproblems, solves each subproblem recursively, and combines their solutions.

Decrease and Conquer: Reduces the problem to a single smaller subproblem and extends its solution.

Examples:

Insertion Sort
Depth-First Search (DFS)
Breadth-First Search (BFS)

2. Variations of Decrease and Conquer Variations:

Decrease by a Constant: The size of the instance is reduced by the same constant on each iteration.

Examples: Insertion Sort, DFS, BFS, Topological Sorting, Permutations/Subsets Generation.

Decrease by a Constant Factor: The size of the instance is reduced by the same constant factor on each iteration.

Examples: Binary Search, Fake-Coin Problems, Russian Peasant Multiplication.

Variable Size Decrease: The size-reduction pattern varies from one iteration to another.

Examples: Computing Median, Interpolation Search, Euclid's Algorithm.

3. Insertion Sort Concepts:

Description: A sorting technique where the sorted array is built one item at a time by inserting elements in their correct position.

Steps:

Start with the second element and compare it with the previous elements.

Insert the element in its correct position by shifting other elements if necessary.

Repeat the process for all elements.

```
void insertionSort(int arr[], int n) {
  for (int i = 1; i < n; ++i) {
    int key = arr[i];
    int j = i - 1;

    while (j >= 0 && arr[j] > key) {
        arr[j + 1] = arr[j];
        j--;
    }
    arr[j + 1] = key;
}
```

Analogy: Imagine arranging a deck of cards. You pick one card at a time and place it in the correct position among the already sorted cards.

Depth-First Search (DFS) (STACK)

Concepts:

Description: A graph traversal algorithm that explores as far as possible along each branch before backtracking.

Steps:

Visit an adjacent unvisited vertex, mark it as visited, display it, and push it onto a stack.

If no adjacent vertex is found, pop a vertex from the stack.

Repeat until the stack is empty.

Complexity Analysis:

Time Complexity: O(V + E), where V is the number of vertices and E is the number of edges.

Space Complexity: O(V), due to the extra visited array needed.

Rules:

Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.

If no adjacent vertex is found, pop up a vertex from the stack.

Repeat Rule 1 and Rule 2 until the stack is empty.

5. Breadth-First Search (BFS) (QUEUE) Concepts:

Description: A graph traversal algorithm that explores the neighbor nodes first before moving to the next level neighbors.

Steps:

Visit an adjacent unvisited vertex, mark it as visited, display it, and insert it into a queue.

If no adjacent vertex is found, remove the first vertex from the queue.

Repeat until the queue is empty.

Complexity Analysis:

Time Complexity: O(V + E), where V is the number of vertices and E is the number of edges.

Rules:

Visit the adjacent unvisited vertex. Mark it as visited. Display it. Insert it in a queue.

If no adjacent vertex is found, remove the first vertex from the queue.

Repeat Rule 1 and Rule 2 until the queue is empty.